defined in [2] is really K_i evaluated at crack initiation of the reinforced matrix. For asbestos–cements K_i is a material property but K_c evaluated at *maximum load* is not because its magnitude increases with specimen size as shown in [3] and noted by Dr Petersson. Moreover, the maximum load K_c does not correspond to the plateau K_R of the K_R-curve. Secondly, it is stated in [2, 3] that $K_c^2 = EG_c$ only when both K_c and G_c refer to *crack initiation*. If G_c is measured from the total work under the load–deflection diagram of a stable three-point bend test on a notched beam, it represents only an *average* specific fracture energy comprising both crack initiation and crack propagation. K_c, calculated from $\sqrt{(EG_c)}$, thus represents only an *average* stress intensity factor. Such a parameter is less useful than a K_R-curve which is able to account for the slow crack growth phenomenon observed even in notched beams with $W = 400$ mm. Thirdly, it seems that E_b instead of E_t should be used in Table II in Dr Petersson's discussion [1] because the three-point notched beams are subjected to bending. Because $E_b = \frac{1}{2}E_t$ the predicted K_c' values from Dr Petersson's analysis and those obtained in [2] will not show the same kind of good agreement as given in his Table II.

In summary, I fully agree with Dr Petersson that the maximum load K_c is too dependent on specimen size to be a useful material property. Unless G_c for crack initiation and crack propagation are identical, which for asbestos–cements they are not, I am not convinced that the true K_c can be simply obtained from $\sqrt{(EG_c)}$, where G_c is obtained from the work of fracture method. To characterize the complete fracture behaviour of asbestos–cements, from initiation, to propagation and to eventual failure I believe that the K_R-curve approach is the most suitable and useful method. We are also currently investigating the G_R-curve approach by considering incremental work dissipation in the fibre pull-out region as the crack slowly extends. In this respect Dr Petersson's Fictitious Crack Model may be useful [5, 6].

References

1. P.-E. PETERSSON, *J. Mater. Sci.* **15** (1980) 2388.
2. Y. W. MAI, *J. Mater. Sci.* **14** (1979) 2091.
3. Y. W. MAI, R. M. L. FOOTE and B. COTTERELL, *Int. J. Cement Composites* **2** (1980) 23.
4. J. C. LENAIN and A. R. BUNSELL, *J. Mater. Sci.* **14** (1979) 321.
5. P.-E. PETERSSON, *Cement and Concrete Research* **10** (1980) 78.
6. P.-E. PETERSSON, *ibid.* **10** (1980) 91.

Received 23 April and accepted 12 May 1980

Y. W. MAI*
Department of Mechanical Engineering, Imperial College, Exhibition Road, London, UK

*On sabbatical leave from the University of Sydney, Sydney, Australia.

On the validity of the Dugdale model for craze zones at crack tips in PMMA

In contrast to previous results [1,2] of interference optical measurements of the craze zone at crack tips in PMMA loaded under Mode-I-conditions, Israel *et al.* [3] report in their recent paper that the Dugdale model is not fully adequate to describe craze geometries in PMMA and from this they suggest a modified craze zone model. They base this hypothesis on their finding that the plastic zone, as calculated from the Dugdale model using constant values of Young's modulus and yield stress and their stress intensity factors for the DCB specimen, is larger (by a factor of about 2.5) than the interference optically measured craze zone. The profile of the craze zone and the Dugdale plastic zone, however, are found to be very similar.

The purpose of this communication is to show that: the Dugdale model describes the profile and size of craze zones in PMMA quite well and gives information about the viscoelastic material behaviour; to examine the discrepancy reported by Israel *et al.*; and to point out some facts suggesting that the authors erred in their determination of K_I.

There is agreement with the authors that in such investigations it is very important to measure

0022–2461/80/092389–06$02.60/0

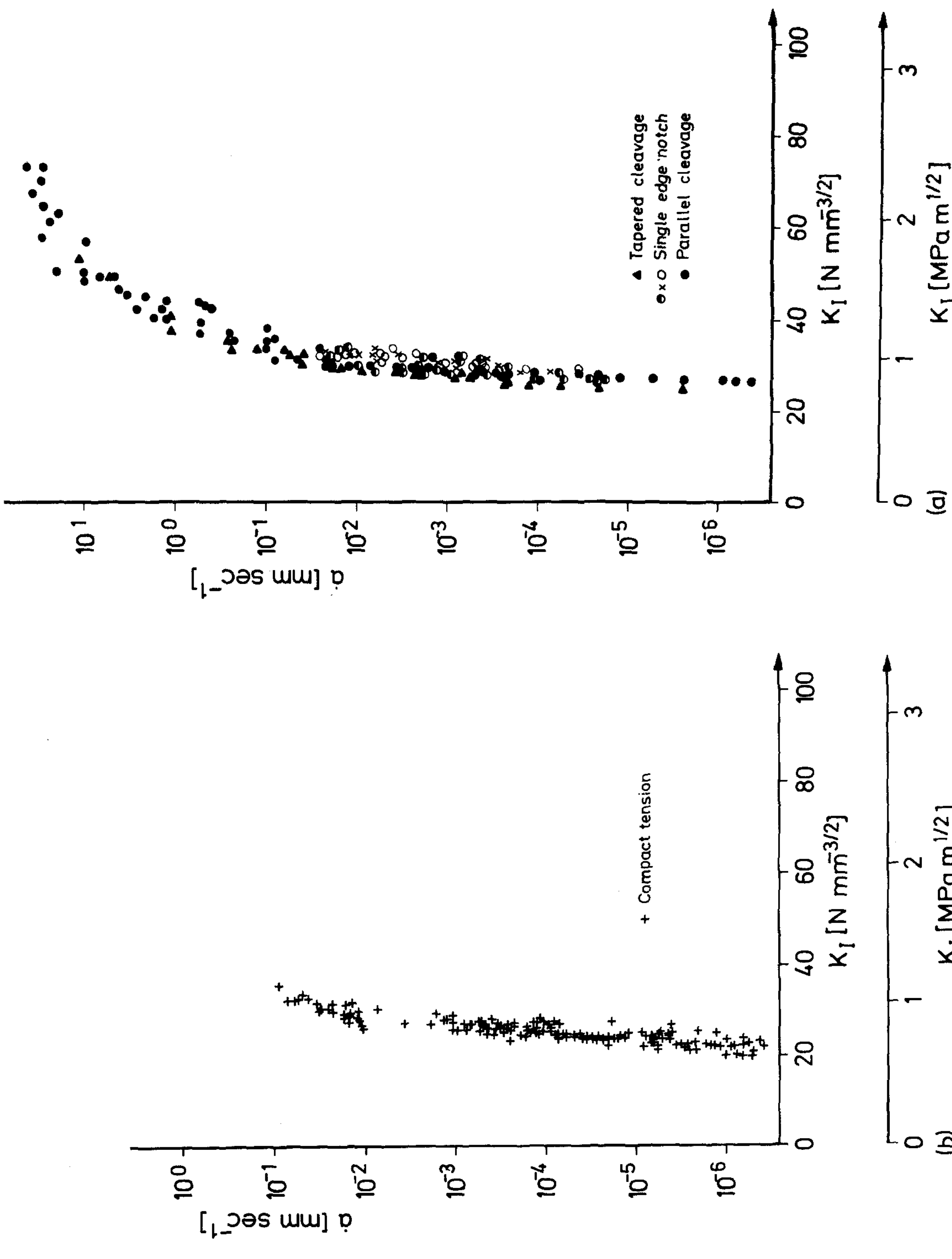

Figure 1 Crack speed $\dot{a}$ as a function of stress intensity factor K_I for PMMA at room temperature. (a) Marshall *et al*. [7], (b) our results.

K_I simultaneously with the record of the interference pattern, in order to characterize the fracture behaviour of the material appropriately. Therefore, in all our interference optical measurements we have used an experimental set-up which allows such synchronous measurements on static [2, 4] as well as on moving cracks [5, 6] to be made. To check our K_I values determined in CT specimens (and not in SEN as mentioned by Israel *et al.* in their Table II) we refer to the paper of Marshall *et al.* [7], who studied crack propagation in PMMA with respect to crack speed $\dot{a}$ and stress intensity factor K_I using three different test methods, namely notched tension, parallel cleavage, and tapered cleavage. Fig. 1 shows their results together with our results [2, 5] of $\dot{a}$–K_I measurements, which are carried out on compact tension specimens. Although the determination of K_I depends upon specimen geometry and loading conditions, there is good agreement between the results of all of the different methods, including the results of Beaumont and Young [8] from double torsion experiments (not shown in Fig. 1). Hence, in this crack speed range there is a unique relationship between the fracture mechanics parameter K_I and crack speed $\dot{a}$ for all types of specimens and loading conditions. It should be mentioned that all our measurements reported in this paper were carried out on a commercial grade cast PMMA with a weight average molecular weight of about 2×10^6, in contrast to $\bar{M}_w = 940\,000$ of the plexiglass used by Israel *et al.* Nevertheless, the results are comparable, because for $\bar{M}_w > 300\,000$ there is only a negligible dependence of deformation and fracture behaviour of PMMA on molecular weight [4, 9].

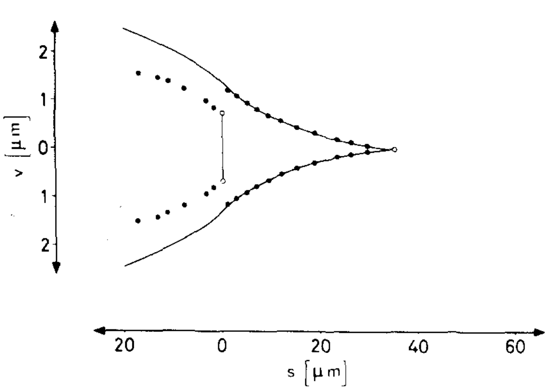

Figure 2 Measured craze and crack opening (points) and fit of Dugdale model (line) for PMMA at $T = 18.5°$ C, $K_I \simeq 22\,N\,mm^{-3/2}$.

When we compare our experimental data on the craze zone with the values predicted by the Dugdale model we obtain very good agreement for the profile (Fig. 2) as well as for the specific dimensions [10]. In addition to the craze profile, we also measured the crack opening by interference optics as can be seen from Fig. 2. The transition of the craze/bulk interface at the crack tip extends as plotted in Fig. 2 by the aid of the Dugdale model, taking into account that a thin layer of craze material remains on both fracture surfaces after cracking (this is easily seen via multicolour on the fracture surface). A continuous transition from craze to crack surface, as given by Israel *et al.* (in their Fig. 11), is probably due to their annealing procedure and is also a hint that in their experiments there was no crack propagation.

In order to determine why Israel *et al.* obtained such a large discrepancy between their experimental results and the predicted values by the Dugdale model, it is necessary to examine their determination of K_I values. The measurements by Israel *et al.* were carried out in a K_I range from 0.3 to $1.75\,MPa\,m^{1/2}$; this would imply crack speeds up to $10\,mm\,sec^{-1}$ as can be seen from Fig. 1. However, in their paper, no remark can be found on an interference experimental set-up for measuring crazes in front of moving cracks at such high crack speeds. Moreover, in the range of K_I values belonging to these high crack speeds, e.g. $K_I = 1.67\,MPa\,m^{1/2}$ (Table I in their paper), the authors find crack initiation. From this discrepancy we estimate their K_I determination to be in error by a factor of about 2 to 3. In checking the K_I determination of Israel *et al.*, we repeated their comparison with results of other formulas given in their Table I. By calculating K_I as a function of DCB beam deflection by different methods [12–15], our results are in contradiction to those of Israel *et al.* in a way that the values from [12–15] together with that of a further model [16] are in the same range, whilst the value given by the formula of Israel *et al.* is too large by a factor of about 3.

It should be mentioned that in the latter calculation we used their polynomial expression for the a/W dependence of the measured compliance and

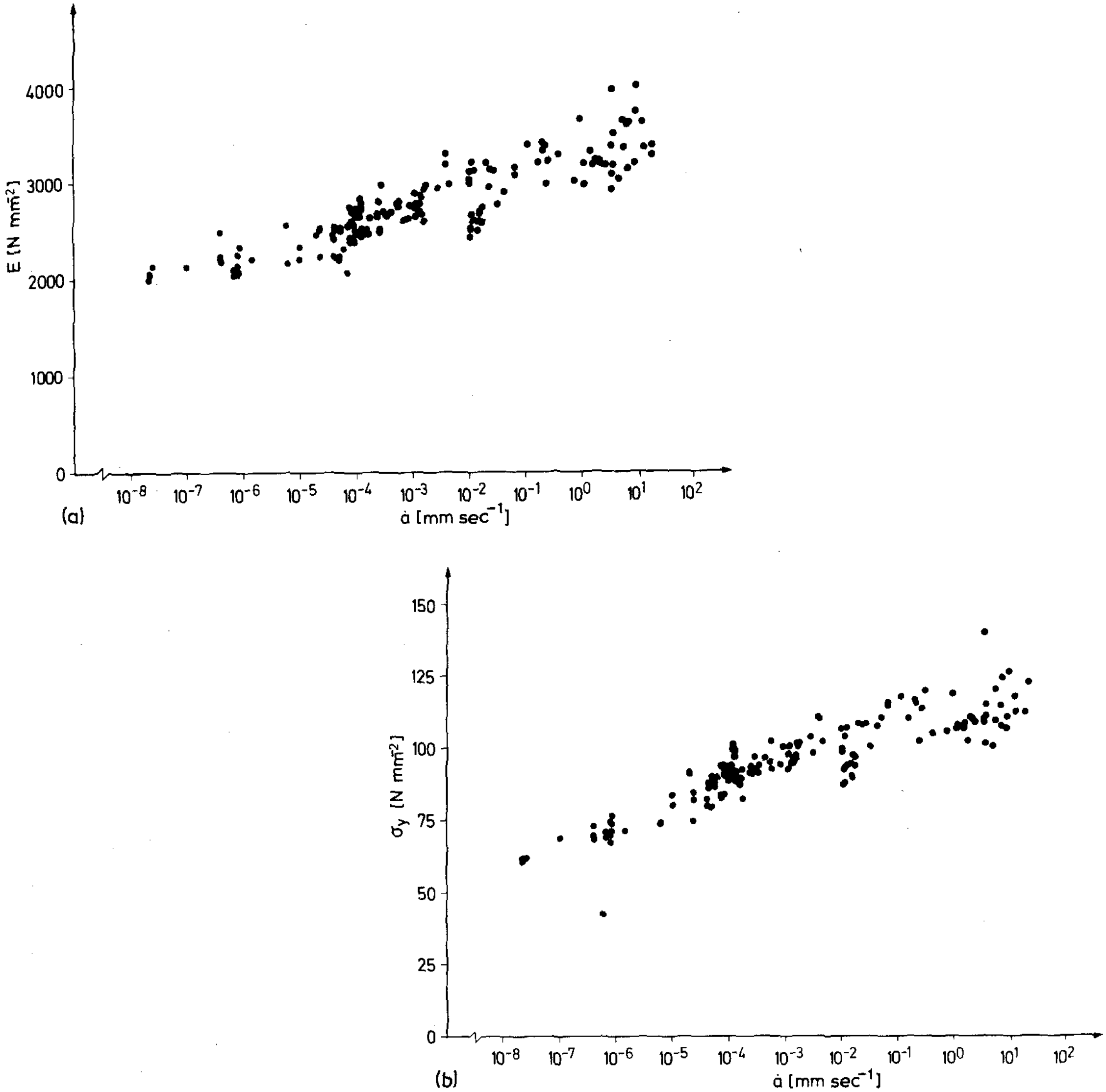

Figure 3 (a) Young's modulus E, and (b) yield stress σ_y as function of crack speed $\dot{a}$ for PMMA at room temperature.

not the one plotted in their Fig. 6, which differs by orders of magnitude from the polynomial expression.

For the calculation of the Dugdale plastic zone size in addition to an accurate K_I-value, Young's modulus (E) and yield stress (σ_{ys}) must be known. These mechanical parameters are time-dependent in the case of a viscoelastic material like PMMA (see, for example, Williams [17]), and hence at propagating cracks in PMMA a time-dependence, or equivalently, a crack speed-dependence of these "moduli" will be involved. To investigate this subject we performed interference optical measurements at moving cracks using a special experimental set-up [5] including simultaneous K_I determination. Fitting the Dugdale model to the measured craze zones, E and σ_{ys} are found to be dependent on crack speed $\dot{a}$ as shown in Fig. 3. In the investigated crack-speed range, E and σ_{ys} increase from 2000 to 3400 N mm^{-2} and 60 to 120 N mm^{-2}, respectively. It becomes obvious that the values of $E = 3100$ N mm^{-2} and $\sigma_{ys} = 72.3$ N mm^{-2} used by Israel *et al.* are not consistent with each other at any given crack speed.

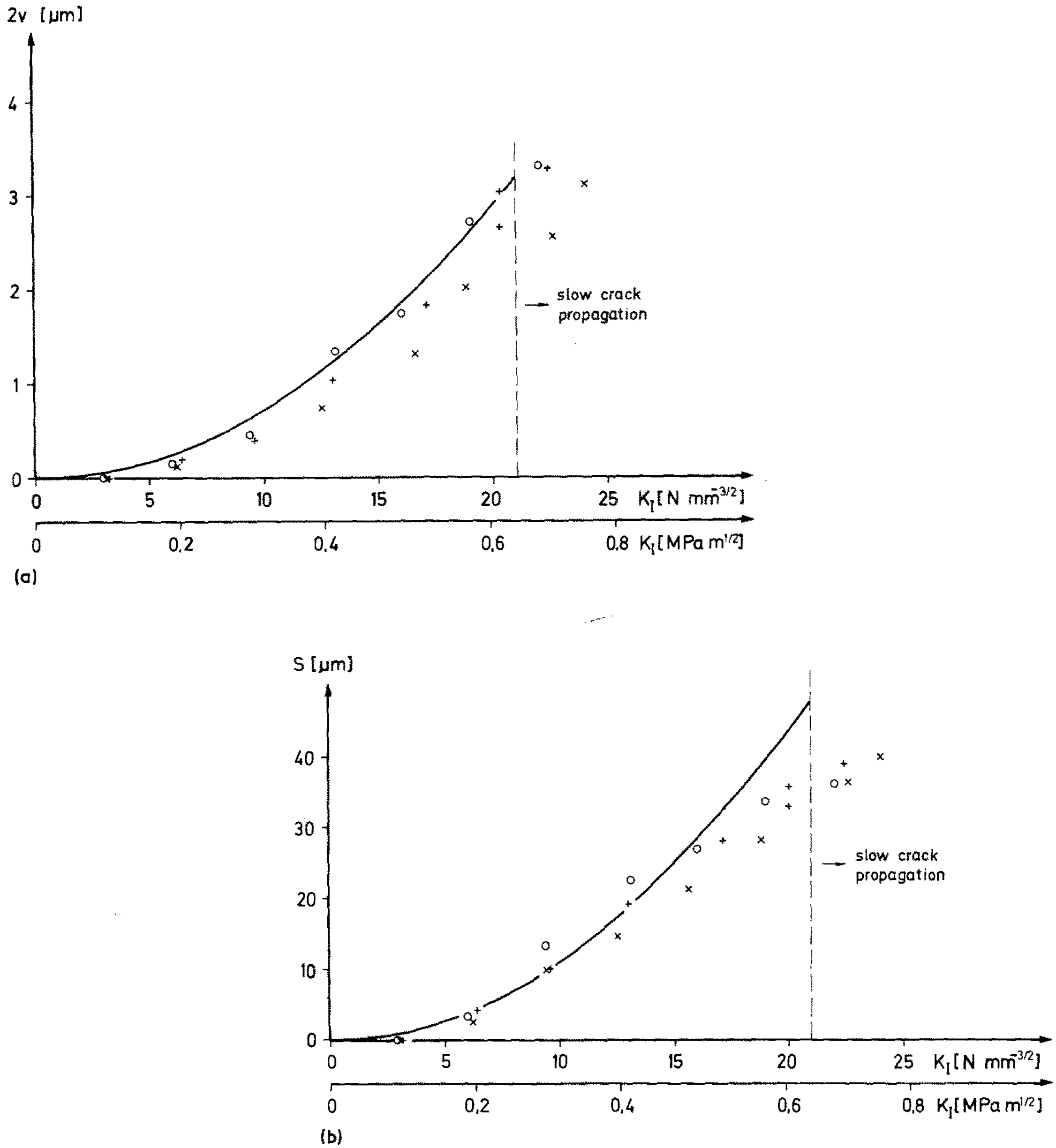

Figure 4 Comparison of (a) maximum craze width $2v$, and (b) craze length s as function of K_I for annealed specimens of PMMA with the behaviour predicted by the Dugdale model (drawn lines).

In order to complete the comparison with the work of Israel *et al.* we also performed experiments on annealed specimens (CT-type) with static cracks. The interference optical results of craze length s and craze width $2v$ at the crack tip are shown in Fig. 4 as a function of the simultaneously measured K_I. To check the validity of the Dugdale model again for this case we calculated (drawn lines) the craze size parameters s and $2v$ using the measured K_I, and values of $E = 2000\,\mathrm{N\,mm^{-2}}$ and $\sigma_{ys} = 60\,\mathrm{N\,mm^{-2}}$ extrapolated from Fig. 3 to $\dot{a} = 10^{-8}\,\mathrm{mm\,sec^{-1}}$ (which should be equivalent to this static case). The agreement between the calculated lines and the experimental results for these static cracks is good. At the onset of slow crack propagation ($K_I > 21\,\mathrm{N\,mm^{-3/2}}$) and at higher values of K_I, the craze length s and craze opening at the crack tip $2v = 2v_c$ are nearly constant with $s \simeq 40\,\mu m$ and $2v_c \simeq 3\,\mu m$, respectively.

In summary, the Dugdale model gives a good description of the profile and size of the craze zones at static and moving crack tips in PMMA and gives the appropriate information of the viscoelastic material behaviour.

Acknowledgement
The experimental work on which this paper is based was financially supported by the Deutsche Forschungsgemeinschaft (DFG).

References
1. H. R. BROWN and I. M. WARD, *Polymer* **14** (1973) 469.
2. G. W. WEIDMANN and W. DÖLL, Proceedings of the Third International Conference on Deformation, Yield and Fracture of Polymers, Cambridge , 1976, (The Plastics and Rubber Institute, London) p. 12.
3. S. J. ISRAEL, E. L. THOMAS and W. W. GERBERICH, *J. Mater. Sci.* **14** (1979) 2128.
4. G. W. WEIDMANN and W. DÖLL, *Colloid and Polymer Sci.* (1976) 205.
5. M. G. SCHINKER and W. DÖLL, Institute of Physics Physics Conference Series no. 47, 1979, edited by J. Harding (The Institute of Physics, Bristol) p. 244.
6. W. DÖLL, M. G. SCHINKER and L. KÖNCZÖL, *Int. J. Fracture* **15** (1979) R 145.
7. G. P. MARSHALL, L. E. CULVER and J. G. WILLIAMS, *Plastics and Polymers*, February (1969) 75.
8. P. W. R. BEAUMONT and R. J. YOUNG, *J. Mater. Sci.* **10** (1975) 1334.
9. W. DÖLL, L. KÖNCZÖL and U. SEIDELMANN, Short Communication presented at the 26th International Symposium on Macromolecules, Mainz, Vol. III (1979) edited by J. Lüderwald and R. Weis p. 1448.
10. G. W. WEIDMANN and W. DÖLL, in "Non-Crystalline Solids", edited by G. H. Frischat (Trans. Tech., Clausthal-Zellerfeld, 1977) p. 606.
11. J. N. GOODIER and F. A. FIELD, in "Fracture in Solids", edited by D. C. Drucker and J. J. Gilman (Interscience, New York, 1963) p. 103.
12. M. F. KANNINEN, *Int. J. Fract.* 9 (1973) 83.
13. P. PARIS and G. SIH, in "Fracture Toughness Testing and Its Applications" (ASTM STP 381, Philadelphia 1965) p. 72.
14. H. TADA, P. PARIS and G. IRWIN, "The Stress Analysis of Cracks Handbook" (Del Research Corporation, Hellertown, Penn., 1973).
15. J. R. RICE, in "Fracture", Vol. II, edited by H. L. Liebowitz (Academic Press, New York, 1968) p. 191.
16. J. RAASCH, *Int. J. Fract. Mech.* 7 (1971) 289.
17. J. G. WILLIAMS, *ibid.* 8 (1972) 393.

Received 11 December 1979
and accepted 11 January 1980

W. DÖLL
U. SEIDELMANN
L. KÖNCZÖL
Fraunhofer-Institut für Werkstoffmechanik,
Rosastrasse 9,
D-7800 Freiburg,
F. R. Germany

A comment on "On the validity of the Dugdale model for craze zones at crack tips in PMMA"

In a publication which investigated the applicability of the Dugdale [1] model for describing craze profiles in PMMA, we concluded that this model was not fully adequate for this purpose [2]. In contrast to these findings, other investigators report that the plastic zone profile described by this model correlates well within the measured craze profiles [3–10]. One of these groups, Döll, Seidelmann and Könczöl [10] are commenting on our publication. They claim that we have errors in both the technique of determining K_I and in the choice of yield strength (σ_{ys}) and elastic modulus (E) used to evaluate the corresponding Dugdale plastic zone profile.

After carefully reviewing our technique for determining K_I it was found that in the preparation of manuscript for the original publication [2] we made the inexcusable mistake of not converting the polynomial expression for the compliance and its derivative to the appropriate S.I. units. Unfortunately, these expressions misled Döll *et al.* [10] in their attempt to reproduce our calculations for comparing our method of determining K_I with those developed by other investigators for the same sample geometry. The corrected expressions for, respectively, Equations 12 (also as an inset on Fig. 6) and 13 in [1] are in metres per Newton

 0022–2461/80/092394–03$02.30/0